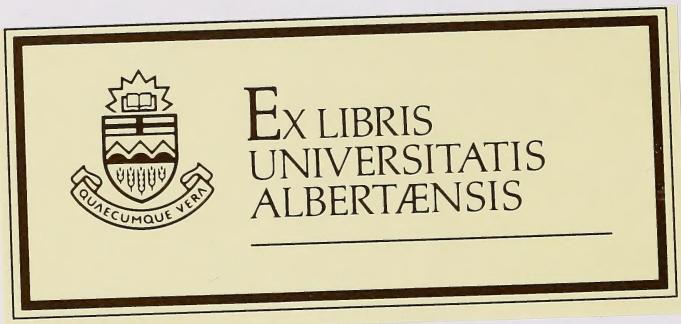


June 1995



Mathematics 30

Grade 12 Diploma Examination



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June 1995

Mathematics 30

Grade 12 Diploma Examination

Description

Time: 2.5 h.

You may take an additional 0.5 h to complete the exam.

Total possible marks: 70

This is a **closed-book** examination consisting of **three** parts:

Part A

has 40 multiple-choice questions each with a value of one mark.

Part B

has 9 numerical-response questions each with a value of one mark.

Part C

has 4 written-response questions for a total of 21 marks.

A tear-out formula sheet and a *z*-score page are included in this booklet.

All graphs on this examination are computer-generated.

Instructions

- Fill in the information required on the answer sheet and the examination booklet as directed by the presiding examiner.
- You are expected to provide your own scientific calculator.
- Carefully read the instructions for each part before proceeding.
- The presiding examiner will collect your answer sheet and examination booklet and send them to Alberta Education.
- Do not fold the answer sheet.

Note: *The perforated pages at the back of this booklet may be torn out and used for your rough work. No marks will be given for work done on the tear-out pages.*

Part A: Multiple Choice

40 Questions

Instructions

- Consider all numbers used in the questions to be **exact real** numbers and not the result of a measurement.
- Read each question carefully and decide which of the choices **best** completes the statement or answers the question.
- Locate that question number on the separate answer sheet provided and fill in the circle that corresponds to your choice.

Example

This diploma examination is for the subject of

- A. biology
- B. physics
- C. chemistry
- D. mathematics

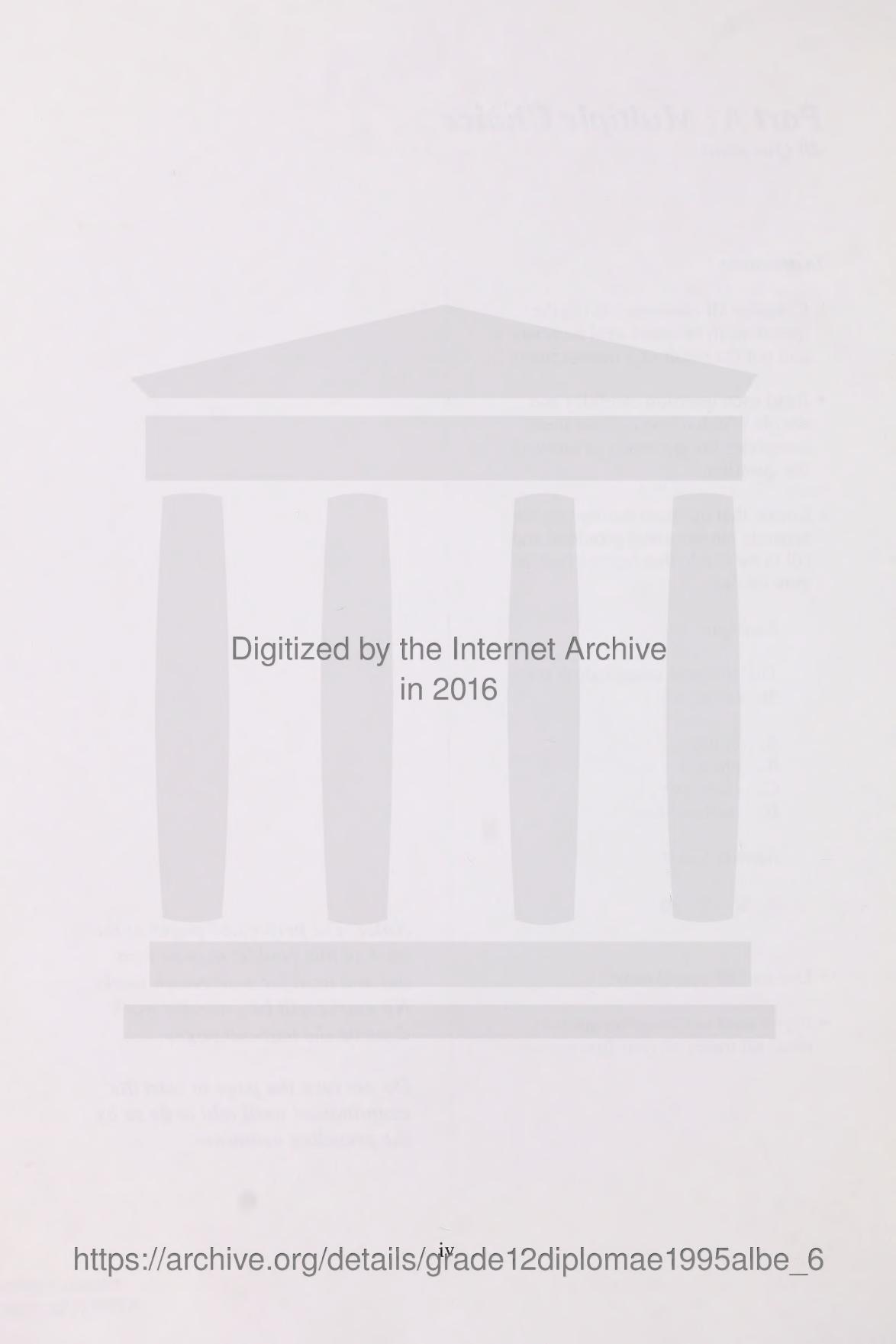
Answer Sheet

(A) (B) (C) ()

- **Use an HB pencil only.**
- If you wish to change an answer, erase **all** traces of your first answer.

Note: The perforated pages at the back of this booklet may be torn out and used for your rough work. **No marks** will be given for work done on the tear-out pages.

Do not turn the page to start the examination until told to do so by the presiding examiner.

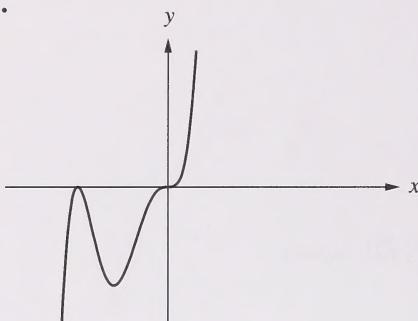
A faint, semi-transparent watermark of a classical building with four columns and a triangular pediment is visible across the page.

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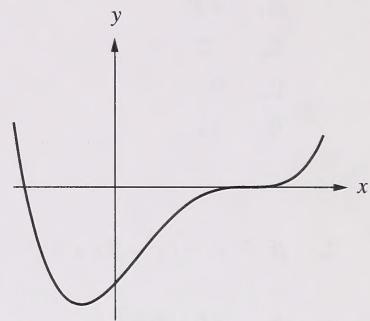
1. If the polynomial $P(x) = (x + 2)(x^2 + 2x - 1) + k$ is divisible by $(x + 4)$, then the value of k is
- A. -4
B. -2
C. 0
D. 14
2. If $P(x) = (x^2 - 2x + 5)(3 - 2x)$, then $P(\sqrt{2})$ equals
- A. $13 - 20\sqrt{2}$
B. $29 - 20\sqrt{2}$
C. $15 - 10\sqrt{2}$
D. $37 - 20\sqrt{2}$
3. If $P(x) = (x + 3)Q(x) + R$, then $P(-3)$ must equal
- A. $Q(3)$
B. $Q(-3)$
C. $-R$
D. R

4. If all of the x -intercepts are shown, then which of the following could represent the graph of a fifth-degree integral polynomial function?

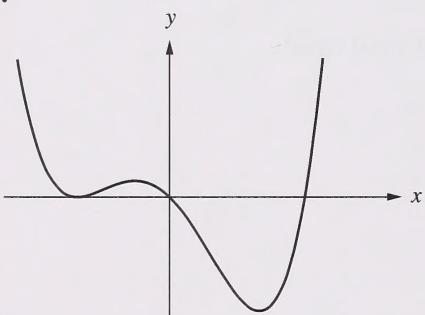
A.



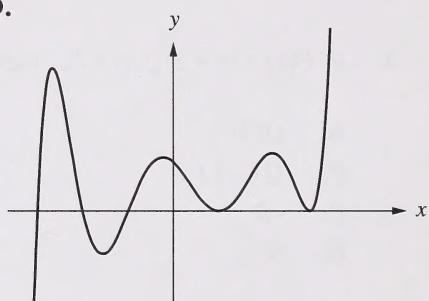
B.



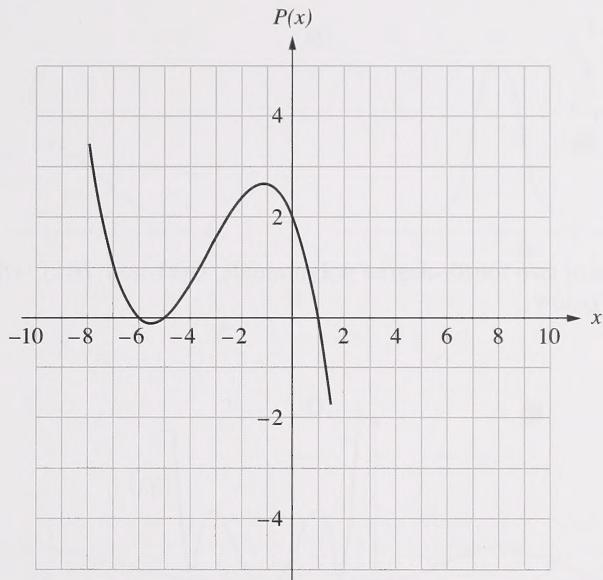
C.



D.



5. The graph of $P(x)$ is shown below.



If $T(x) = P(x) + 2$, then the true statement about the graph of $T(x)$ is that the

- A. y -intercept is 0
- B. y -intercept is 4
- C. x -intercepts are $-4, -3, 3$
- D. x -intercepts are $-8, -7, -1$

6. An example of a polynomial function is

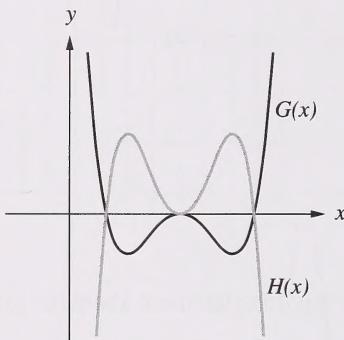
A. $P(x) = \sqrt{3}x^3 - 3x^2 + 4$

B. $P(x) = 2x^{-2} + x + 3$

C. $P(x) = \sqrt{3x^3} - 2x + 7$

D. $P(x) = 2^x + 4$

7. There are three factors of $3x^3 - 2x^2 - 17x - 12$. Two of the factors are $x - 3$ and $x + 1$. What is the third factor?
- A. $3x - 4$
B. $3x + 4$
C. $-3x + 4$
D. $-3x - 4$
8. The graphs of two fourth-degree polynomials $G(x)$ and $H(x)$, where $H(x) = -2G(x)$ are shown below.



Three observations about the graphs are:

- I. $G(x)$ and $H(x)$ have the same zeros
- II. the domains of $G(x)$ and $H(x)$ are the same
- III. the ranges of $G(x)$ and $H(x)$ are the same

Which of the observations are **true**?

- A. Observation I only
B. Observation III only
C. Observations I and II only
D. Observations II and III only

9. If the same scale is used, then which of the following normal curves represents the data with the smallest standard deviation?

A.



B.



C.



D.



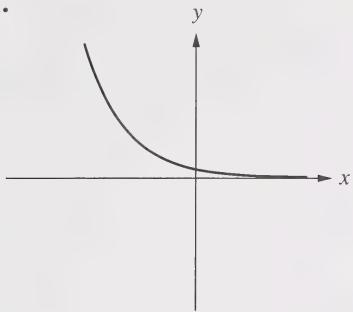
10. The waiting times for patrons entering an auditorium are normally distributed with a mean of 30 min and a standard deviation of 5 min. If 2500 people enter the auditorium, then the number of people having to wait longer than 38 min is

- A. 137
B. 1113
C. 1387
D. 2363

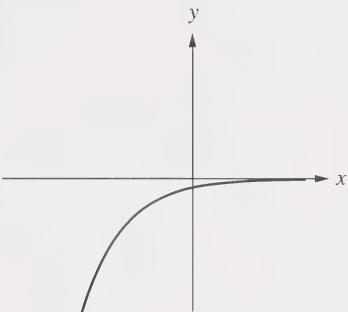
11. The raw scores on a test are normally distributed with a standard deviation of 17. If Graham's raw score on the test is 1.4 standard deviations above the mean, then his raw score exceeds the mean by
- A. 5.7
B. 7.1
C. 23.8
D. 41.9
12. In a normal distribution, the percentage of the population that lies between $\mu - \sigma$ and $\mu - 2\sigma$ is
- A. 13.59%
B. 47.72%
C. 68.26%
D. 81.85%
13. The results of a test are normally distributed with a mean of 36. If 2.28% of the students scored less than 28, then the percentage of students with scores greater than 38 is
- A. 19.15%
B. 28.57%
C. 30.85%
D. 38.07%

14. The graph of $x = \log_3(y)$ is represented by

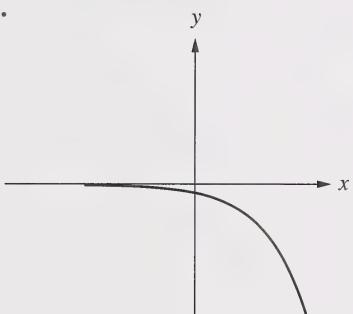
A.



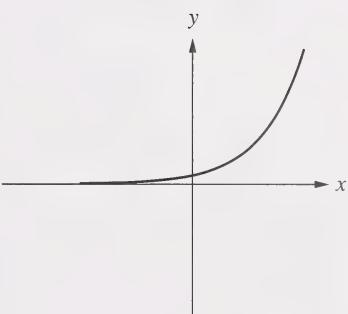
B.



C.



D.



15. An exponential form of $z = \log_a(x) - \log_a(y)$ is

A. $a^z = \frac{x}{y}$

B. $z^a = \frac{x}{y}$

C. $a^z = x - y$

D. $z^a = x - y$

- 16.** Given that a , b , and $c \in R$, and $a > 0$, $b \neq 0$, which of the following functions is an exponential function?
- A. $f(x) = x^a - c$
- B. $f(x) = x^a + b$
- C. $f(x) = a^{bx + c}$
- D. $f(x) = ax + b^c$
- 17.** If $\log_5 100 - \log_5 m = 2$, then the value of m is
- A. 4
- B. 10
- C. 75
- D. 98
- 18.** If $\log_5(2x + 1) + \log_5(x - 1) = 1$, then x is
- A. $-\frac{3}{2}$ or 2
- B. $\frac{3}{2}$ or -2
- C. $-\frac{3}{2}$
- D. 2

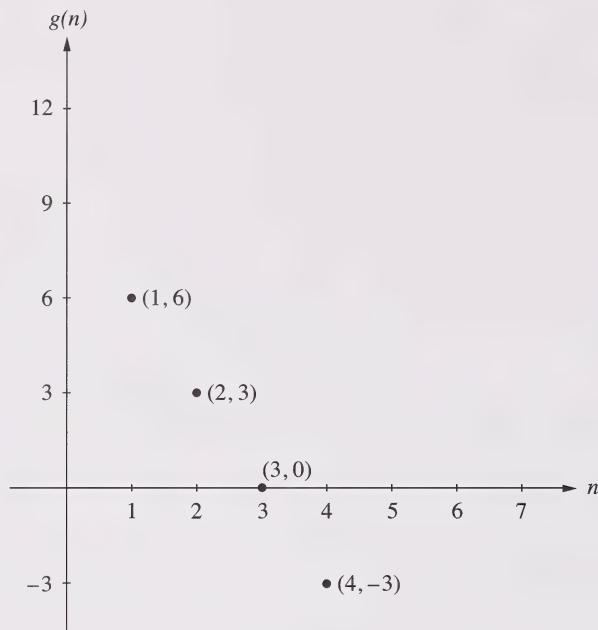
- 19.** A sequence is defined recursively as

$$\begin{aligned}t_1 &= -3 \\t_n &= 2t_{n-1} + 5, \quad n \geq 2\end{aligned}$$

The value of t_5 is

- A. 5
 - B. 13
 - C. 15
 - D. 27
- 20.** A bouncing ball regains 75% of its height after each bounce. If the ball is originally dropped from a height (h), its maximum height after n bounces is represented by
- A. $h(n) = h(0.25)^n$
 - B. $h(n) = h(0.75)^n$
 - C. $h(n) = h(0.25)^{n-1}$
 - D. $h(n) = h(0.75)^{n-1}$

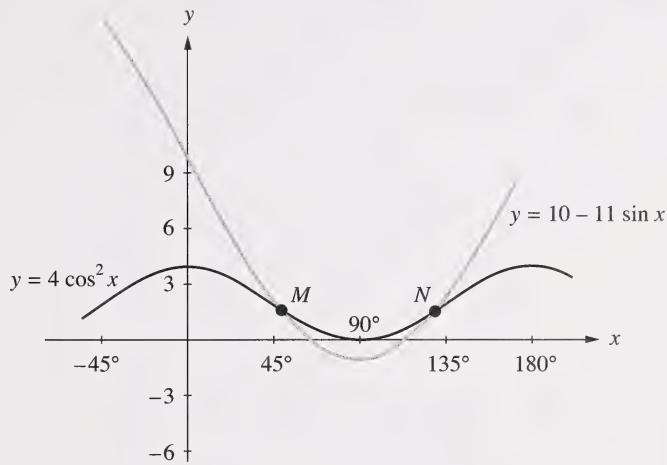
Use the following information to answer the next question.



21. The common difference of the sequence $g(n)$ where $n \in N$ is

- A. -3
- B. -1
- C. 1
- D. 3

22. The partial graphs of $y = 4 \cos^2 x$ and $y = 10 - 11 \sin x$ intersect at points M and N as shown.



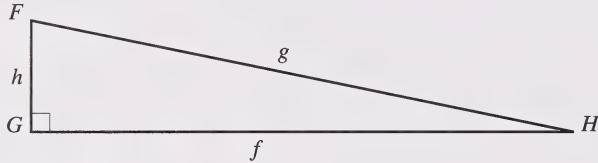
The x -coordinates of points M and N are solutions of

- A. $4 \cos^2 x = 0$
B. $10 - 11 \sin x = 0$
C. $4 \cos^2 x - 11 \sin x + 10 = 0$
D. $4 \cos^2 x + 11 \sin x - 10 = 0$
23. The solution of the trigonometric equation $(2 \sin \theta + 1)(\cos \theta) = 0$, $0 \leq \theta \leq \frac{\pi}{2}$, is
- A. 0
B. $\frac{\pi}{6}$
C. $\frac{\pi}{2}$
D. $\frac{\pi}{3}$

24. The function $f(\theta) = (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2$ may be written as

- A. $f(\theta) = 2$
- B. $f(\theta) = 2 \sin \theta$
- C. $f(\theta) = 2 \sin^2 \theta$
- D. $f(\theta) = 2 - 2 \sin \theta \cos \theta$

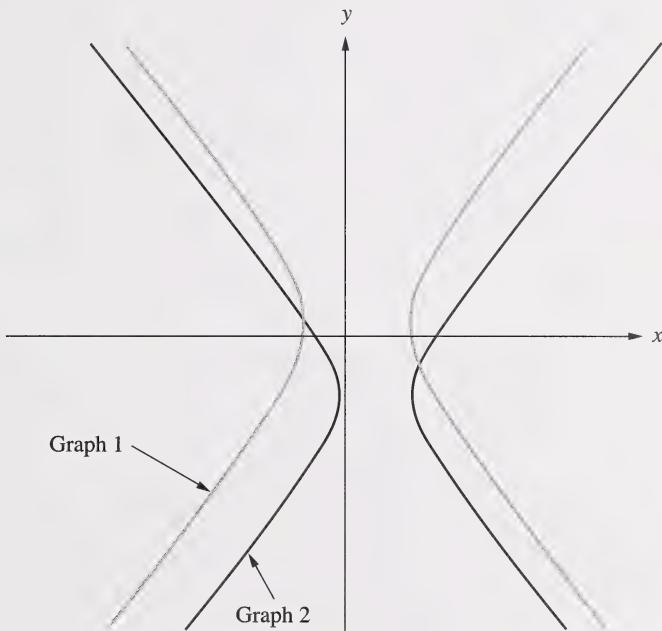
Use the following information to answer the next question.



25. The expression $\sin(90^\circ + \angle F)$ is equal to

- A. $\frac{f}{g}$
- B. $\frac{h}{g}$
- C. $\frac{g+f}{g}$
- D. $\frac{h+f}{g}$

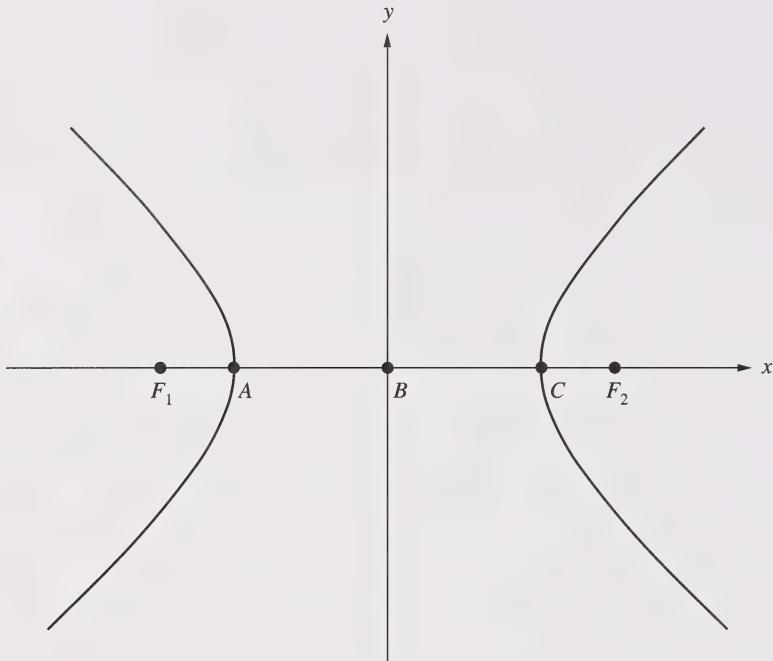
26. Marcia was doing a mathematics lab to investigate the effects of changing the coefficients A , C , D , E , and F in the equation of the quadratic relation defined by $Ax^2 + Cy^2 + Dx + Ey + F = 0$. Marcia entered values for A , C , D , E , and F into the computer and obtained graph 1. She changed values of only two of these coefficients to obtain graph 2.



The two coefficients Marcia changed to obtain graph 2 could have been

- A. A and C
- B. A and D
- C. C and E
- D. D and E

27. Let P be any point on the hyperbola shown below. The foci of the hyperbola are F_1 and F_2 . \overline{PF}_1 is the distance from P to F_1 , and \overline{PF}_2 is the distance from P to F_2 .



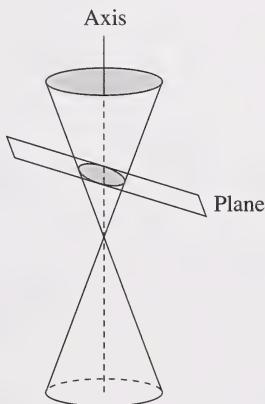
If the hyperbola is symmetric with respect to the y -axis, then $|\overline{PF}_1 - \overline{PF}_2|$ equals

- A. \overline{AC}
- B. \overline{AF}_2
- C. \overline{BC}
- D. \overline{BF}_2

28. If $Ax^2 + Cy^2 + Dx + Ey + F = 0$ defines a non-degenerate parabola, then which of the following restrictions can apply to the values of the coefficients?
- A. $C = 0$
 - B. $A = 0, D = 0$
 - C. $A = 0, C = 0$
 - D. $C = 0, E = 0$

Use the following information to answer the next question.

A circular conical surface is cut by a plane that is not perpendicular to the axis of the surface, as shown.



29. The eccentricity of the conic section formed by the intersection of the plane and the circular conical surface is
- A. 0
 - B. 1
 - C. greater than 1
 - D. greater than 0 but less than 1

30. The eccentricity of a quadratic relation is $\frac{1}{5}$. If the directrix is $x = -2$ and the focus is $F(3, -2)$, then the equation that describes this quadratic relation is
- A. $\frac{1}{5} = \frac{\sqrt{(x - 3)^2 + (y + 2)^2}}{\sqrt{(x + 2)^2 + (y + 2)^2}}$
- B. $\frac{1}{5} = \frac{\sqrt{(x + 2)^2 + (y + 2)^2}}{\sqrt{(x - 3)^2 + (y + 2)^2}}$
- C. $\frac{1}{5} = \frac{\sqrt{(x - 3)^2 + (y + 2)^2}}{\sqrt{(x + 2)^2}}$
- D. $\frac{1}{5} = \frac{\sqrt{(x + 2)^2}}{\sqrt{(x - 3)^2 + (y + 2)^2}}$
31. If the points $(-1, -1)$ and $(-4, 4)$ are on the parabola defined by $y^2 + Dx + F = 0$, then D and F respectively are
- A. 5 and 4
- B. 5 and -4
- C. -5 and 4
- D. -5 and -4
32. A “word” is any sequence of one or more letters. If the letters **A E K S T** form a 5-letter “word” with no repeated letters, then the probability that the word is **SKATE** is
- A. $\frac{1}{5!}$
- B. $\frac{2}{5!}$
- C. $\frac{1}{5}$
- D. $\frac{2}{5}$

33. The number of different 5-letter permutations of all the letters in the word **APPLE** is
- A. 5
B. 10
C. 60
D. 120
34. The number of ways of arranging six candles of different colours on the perimeter of a circular birthday cake is
- A. $5!$
B. $6!$
C. $\frac{5!}{2}$
D. $\frac{6!}{2}$
35. Private automobile licence plates contain three letters followed by three numbers. One example is JJM030. If there are no other constraints, the number of plates that could be manufactured is
- A. $26! \times 10!$
B. $26^3 \times 10^3$
C. $26! \times 25! \times 24! \times 10! \times 9! \times 8!$
D. $26 \times 25 \times 24 \times 10 \times 9 \times 8$

- 36.** Eight students volunteer to help out at the school dance. Two students are needed to check coats, two other students are needed to sell soft drinks, and the remaining four are needed for cleanup. The number of different ways these students can be assigned to their duties is
- A. 16
B. 128
C. 336
D. 420
- 37.** The schedule of a softball league consisting of five teams requires each team to play three games against every other team. The number of games that must be scheduled for this league is
- A. 15
B. 30
C. 45
D. 60
- 38.** The value of ${}_nP_2$ is
- A. $\frac{n}{2}$
B. $\frac{n!}{2}$
C. $n^2 - n$
D. $\frac{n^2 - n}{2}$

- 39.** The fifth-degree term in the expansion of $(y - 3)^7$ is
- A. $189y^5$
B. $-189y^5$
C. $21y^5$
D. $-21y^5$
- 40.** You are a group leader at a summer camp. You must assign 10 children to two activities: swimming or horseback riding. There must be five children in each activity. Three of the children must be assigned to swimming because they are allergic to horses and one child must be assigned to horseback riding because he cannot swim. If there are no other restrictions, how many ways can the children be assigned to the two activities?
- A. 15
B. 30
C. 180
D. 225

You have now completed Part A. Proceed directly to Part B.

Part B: Numerical Response

9 Questions

Instructions

- Consider all numbers used in the questions to be **exact positive real** numbers and not the result of a measurement.
- Read each question carefully.
- Record your answer on the answer sheet provided by writing it in the boxes and then filling in the corresponding circles.
- **Enter the first digit of your answer in the left-hand box and leave any unused boxes blank.**
- Use an HB pencil only.
- If you wish to change an answer, erase **all** traces of your first answer.

Sample Questions and Solutions

Correct to the nearest tenth of a radian, 40° is equal to _____ rad.

$$40^\circ = 0.6981317008 \dots \text{rad}$$

Record 0.7 on the →

0	.	7
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answer sheet

0	.	7
---	---	---

<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
0	0	0
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7
8	8	8
9	9	9

For the arithmetic series $-8 + (-5) + (-2) + \dots + (85)$, the number of terms is _____.

$$85 = -8 + (n - 1)(3)$$

$$93 = 3n - 3$$

$$n = 32$$

Record 32 on the →

3	2	
---	---	--

answer sheet

3	2	
---	---	--

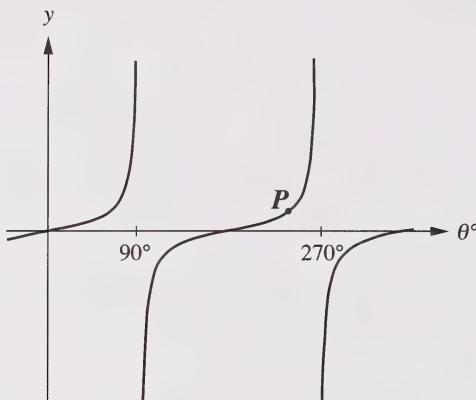
<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
0	0	0
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7
8	8	8
9	9	9

Start Part B immediately.

- 1.** The minimum degree of a polynomial function that has two equal positive real zeros and two distinct negative real zeros is _____.
(Record your answer on the answer sheet.)
- 2.** The roots of the equation of a third-degree polynomial function $P(x) = ax^3 + bx^2 + cx + d$ are $-2, 1$, and 3 . If the graph of the polynomial function passes through $(0, 12)$, then, correct to the nearest tenth, the value of the **constant**, d , is _____.
(Record your answer on the answer sheet.)
- 3.** The point $(9, -2)$ lies on the graph of $y = \log_b x$. Correct to the nearest hundredth, the value of b is _____.
(Record your answer on the answer sheet.)
- 4.** In a supermarket, tins of tuna are stacked in the form of a pyramid. The bottom layer contains 20 tins, and there are two fewer tins on each succeeding layer. If the top layer contains four tins, then the number of tins of tuna in the display is _____.
(Record your answer on the answer sheet.)

- 5.** The sum of the arithmetic sequence $30, 25, 20, \dots t_n$ is -35 . The number of terms in this sequence is _____.
(Record your answer on the answer sheet.)
- 6.** The general term of a sequence is $t_n = 2(3)^{n-1}$. The sum of the first eight terms in this sequence is _____.
(Record your answer on the answer sheet.)
- 7.** Correct to the nearest degree, an angle measure of 2.3 radians is _____ $^{\circ}$.
(Record your answer on the answer sheet.)

- 8.** The point $P(\theta, 4.2)$ lies on the graph of $y = \tan \theta$, $0^\circ \leq \theta \leq 360^\circ$, as shown below.



Correct to the nearest degree, the value of θ at point P is _____ $^\circ$.
(Record your answer on the answer sheet.)

- 9.** There are only seven seats in a van. One of the seats is modified to fit only a baby and one is the driver's seat. The number of ways that six adults and a baby can be legally seated in the van if only four of the adults are licenced to drive is _____.
(Record your answer on the answer sheet.)

You have now completed Part B. Proceed directly to Part C.

Part C: Written Response

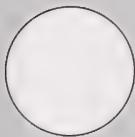
4 Questions

Instructions

- Consider all numbers used in the question to be **exact real** numbers and not the result of a measurement.
- Read each question carefully.
- Write your answers in the examination booklet as neatly as possible.
- For full marks, your answers **must show all** pertinent explanations, calculations, and formulas.
- Your answers **should be** presented in a well-organized manner using complete sentences for a written response, and correct units for a numerical response.

Note: *The perforated pages at the back of this booklet may be torn out and used for your rough work. No marks will be given for work done on the tear-out pages.*

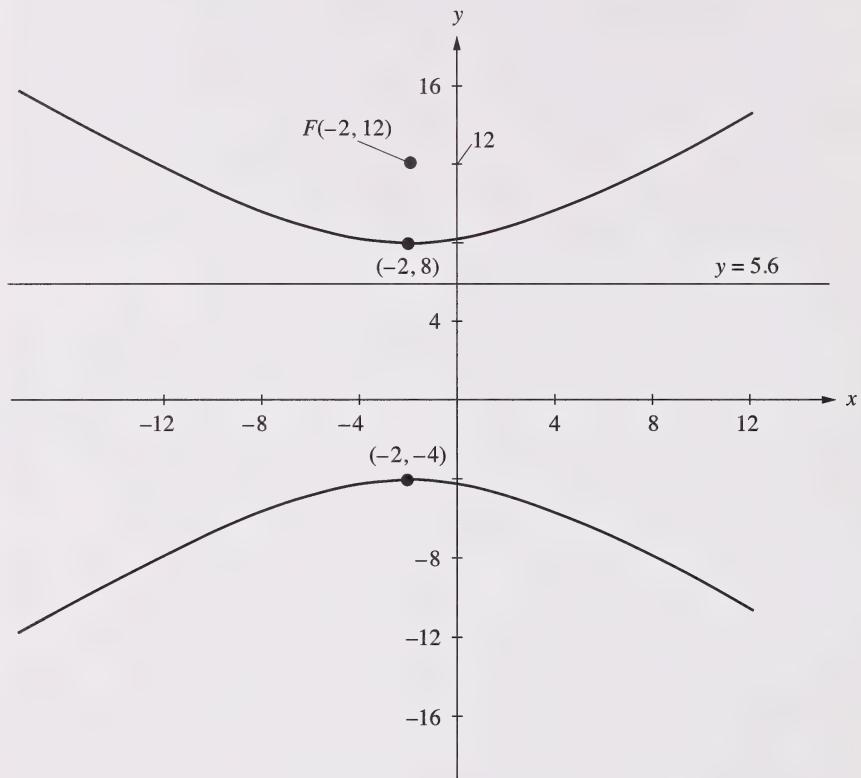
Start Part C immediately.



1. Jean put \$2000 under her mattress for safekeeping. Inflation is expected to halve the value of the money every 15 years. Without using a calculator, Jean estimates that when she retires in 40 years her money will be worth between \$250 and \$500 in today's dollars.
 - Explain a process Jean could have used to make her **estimate**.

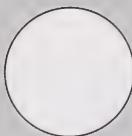
- Correct to the nearest cent, determine the actual expected value of Jean's \$2000, measured against today's dollar when she retires in 40 years.

2. In the hyperbola shown below, one focus is at $F(-2, 12)$. The corresponding directrix is $y = 5.6$. The points $(-2, 8)$ and $(-2, -4)$ lie on the graph of a hyperbola.



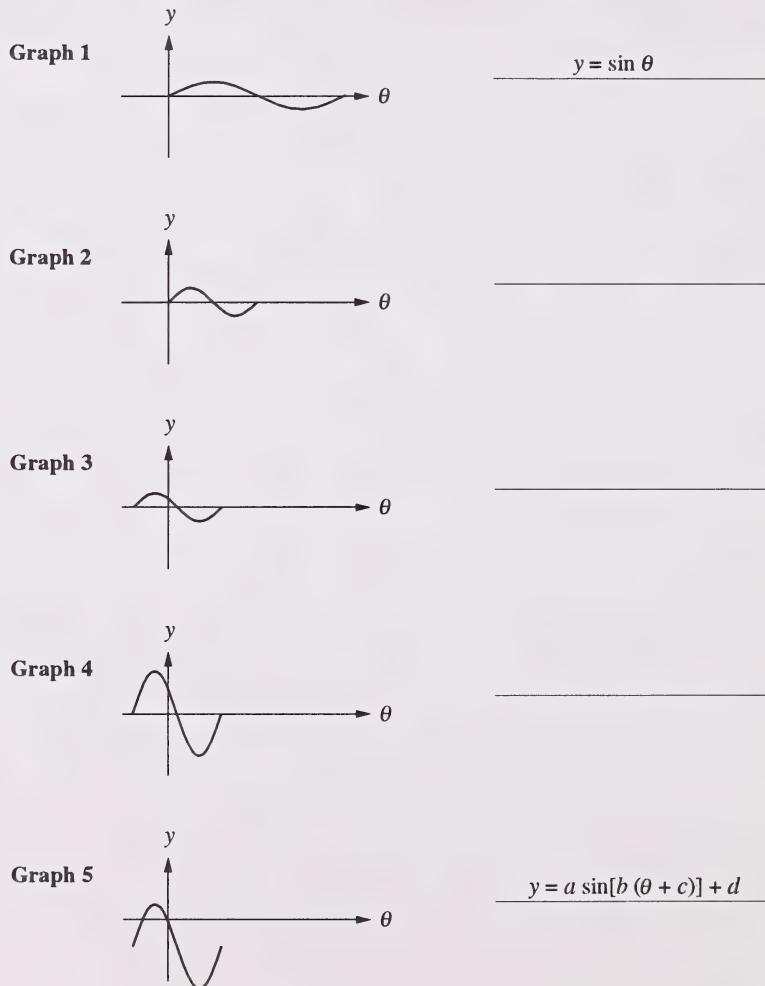
- Another focus and directrix will also define this hyperbola. Determine this focus and its corresponding directrix.

- Explain **how** you could decide if the point $P(-15, 13)$ lies on the hyperbola.



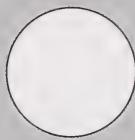
3. A student was demonstrating the combined effects of the parameters a , b , c , and d in the equation $y = a \sin [b(\theta + c)] + d$ on the graph of $y = \sin \theta$. The student sketched one period of the sine graph and then showed the effect of changing one parameter at a time.

- Below are five graphs. The first one is one period of the graph of $y = \sin \theta$, and the last one is one period of the graph of $y = a \sin [b(\theta + c)] + d$. All graphs are on grids of the same scale. Graph 2 represents the effect of one parameter change on graph 1, graph 3 represents the effect of one parameter change on graph 2, and so on for each graph. Write the general equation beside graphs 2 to 4 to demonstrate which parameter has been introduced.



- Explain the effect of introducing each parameter a , b , c , and d on the graph of $y = \sin \theta$, and describe how these parameters affect the function's domain and range.

(Question 4 is on the next page.)



4. Algebraically, determine the positive value of K such that
- $$\sum_{n=1}^K 4n = 840.$$

*You have now completed the examination.
If you have time, you may wish to check your answers.*

Mathematics 30 Formula Sheet

The following information may be useful in writing this examination.

- The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The distance between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Quadratic Relations

- $e = \frac{|\overline{PF}|}{|\overline{PD}|}$

Trigonometry

- arc length $a = r\theta$
- $\sin^2 A + \cos^2 A = 1$
- $1 + \tan^2 A = \sec^2 A$
- $1 + \cot^2 A = \csc^2 A$
- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\csc A = \frac{1}{\sin A}$
- $\sec A = \frac{1}{\cos A}$
- $\cot A = \frac{\cos A}{\sin A}$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Permutations and Combinations

- ${}_n P_r = \frac{n!}{(n-r)!}$

- ${}_n C_r = \frac{n!}{r!(n-r)!}$

- In the expansion of $(x + y)^n$, the general term is $t_{k+1} = {}_n C_k x^{n-k} y^k$

Sequences and Series

- $t_n = a + (n-1)d$

- $t_n = ar^{n-1}$

- $S_n = \frac{n[2a + (n-1)d]}{2}$

- $S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1$

- $S_n = n \left(\frac{a + t_n}{2} \right)$

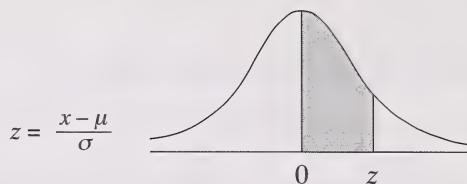
- $S_n = \frac{rt_n - a}{r - 1}, r \neq 1$

Exponential and Logarithmic Functions

- $\log_a mn = \log_a m + \log_a n$

- $\log_a \frac{m}{n} = \log_a m - \log_a n$

- $\log_a m^n = n \log_a m$



Areas under the Standard Normal Curve

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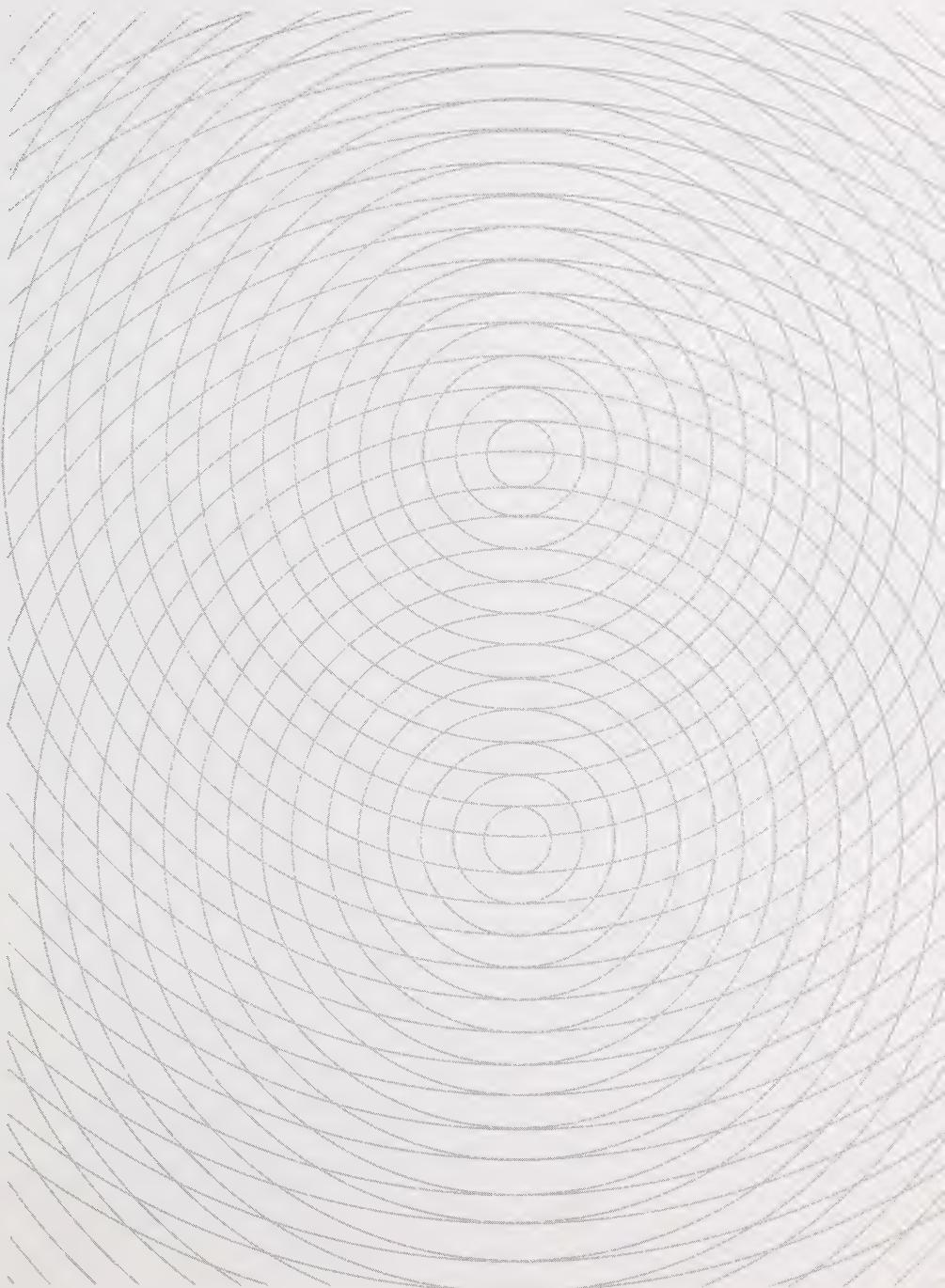
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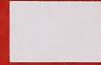


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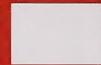
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